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TECHNICAL REPORT NO. 256 TR-256

May 1975

A (k+1)-DECISION TWO-STAGE SELECTION PROCEDURE FOR COMPARING k NORMAL MEANS WITH A FIXED KNOWN STANDARD: THE CASE OF COMMON UNKNOWN VARIANCE

by

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Prepared under contracts

DAHC04-73-C-0008, U. S. Army Research Office - Durham

and

N00014-67-A-0077-0020, Office of Naval Research

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B.

Summary

A (k+1)-decision two-stage selection procedure is proposed for the problem of comparing k normal means with a fixed known standard when the associated populations have a common unknown variance. None or one of the populations is to be selected, the procedure having the property that (1) with probability at least P_0^* (specified), no population is to be selected when the largest population mean is sufficiently less than the standard, and ~~(ii)~~ (2) with probability at least P_1^* (specified), the population with the largest population mean is to be selected when that mean is sufficiently greater than its closest competitor and the standard. Tables to implement the procedure are provided. Applications and generalizations are described.

Some key words: Ranking procedures, selection procedures, two-stage procedures, comparisons with a fixed standard, indifference-zone approach, sampling plans.

1. Introduction

In an earlier paper, Bechhofer and Turnbull (1974), the authors proposed a (k+1)-decision single-stage selection procedure for comparing k normal means with a fixed known standard when the k populations have a common known variance. In the present paper we propose a two-stage procedure for the same problem when the populations have a common unknown variance. Thus the present paper bears the same relationship to our earlier paper as does Bechhofer, Dunnett, and Sobel (1954) to Bechhofer (1954). The reader is referred to our earlier paper for motivation concerning the practical situations which lead us to consider such (k+1)-decision selection problems, and to B-D-S (1954) and Dudewicz (1971) for the rationale associated with adopting a two-stage procedure when the experimenter is faced with situations in which the common variance is unknown.

2. Assumptions

We assume that we have k normal populations Π_i with unknown population means μ_i $(1 \leq i \leq k)$ and a common unknown variance σ^2; there is also a given known standard μ_0 with which the μ_i are to be compared. The ranked values of the μ_i are denoted by $\mu_{[1]} \leq \mu_{[2]} \leq \cdots \leq \mu_{[k]}$. It is assumed that the experimenter has no prior knowledge concerning the pairing of the Π_i with the $\mu_{[j]}$ $(1 \leq i,j \leq k)$; it is further assumed that the experimenter has no prior knowledge concerning how many or which (if any) populations have μ-values which are $\geq \mu_0$.

3. Goal, probability requirement, and procedure

In this section we formulate our goal and probability requirement using the indifference-zone approach, and propose a two-stage procedure which will guarantee this probability requirement.

3.1 The goal

The goal (i.e., objective of the experiment) is:

"To select the population associated with $\mu_{[k]}$ provided that $\mu_{[k]} > \mu_0$; if no population has a μ-value which is $> \mu_0$, then no population is to be selected." (1)

3.2 The probability requirement

It is assumed that prior to the start of experimentation, the experimenter can specify five constants $\{\delta_0^*, \delta_1^*, \delta_2^*; P_0^*, P_1^*\}$ $\{0 < \delta_1^*, \delta_2^* < \infty,\ -\delta_1^* < \delta_0^* < \infty;\ 2^{-k} < P_0^* < 1,\ (1-2^{-k})/k < P_1^* < 1\}$ the values of which are to be based on economic considerations. The specified constants along with the known standard are incorporated into the following probability requirement:

$$\Pr\{\Pi_0\} \geq P_0^* \quad \text{whenever} \quad \mu_{[k]} \leq \mu_0 - \delta_0^*, \tag{2a}$$

$$\Pr\{\Pi_{[k]}\} \geq P_1^* \quad \text{whenever} \quad \begin{cases} \mu_{[k]} \geq \mu_0 + \delta_1^* \\ \text{and} \\ \mu_{[k]} \geq \mu_{[k-1]} + \delta_2^* \end{cases} \tag{2b}$$

where Π_0 ($\Pi_{[k]}$) denotes the event of selecting no population (the population associated with $\mu_{[k]}$).

3.3 The selection procedure

The two-stage procedure which we adopt in this paper, and which guarantees (2a) and (2b), is:

"a) In the first stage take a common number $N_0 > 1$ of observations X_{ij} $(1 \leq i \leq k,\ 1 \leq j \leq N_0)$ from each of the k populations.

b) Calculate $S^2 = \sum_{i=1}^{k} \sum_{j=1}^{N_0} (X_{ij} - \sum_{j=1}^{N_0} X_{ij}/N_0)^2/n$ which is an unbiased estimate of σ^2 based on $n = k(N_0-1)$ degrees of freedom.

c) Enter the appropriate[1/] table with $k, n = k(N_0-1)$, and the specified quantities $\{\delta_0^*, \delta_1^*, \delta_2^*; P_0^*, P_1^*\}$ and obtain a scalar h and a constant c (in the units of the problem). (3)

d) In the second stage, take a common number $N - N_0$ of additional observations from each of the k populations where $N = \max\{[(hS/(\delta_0^* + c))^2] + 1, N_0\}$, and $[x]$ denotes the largest integer less than x.

e) Calculate the k over-all (first-stage plus second-stage) sample means $\bar{X}_i = \sum_{j=1}^{N} X_{ij}/N$ $(1 \leq i \leq k)$ and denote their ranked values by $\bar{X}_{[1]} < \ldots < \bar{X}_{[k]}$.

f) If $\bar{X}_{[k]} < \mu_0 + c$, select no population; if $\bar{X}_{[k]} > \mu_0 + c$, select the population that produced $\bar{X}_{[k]}$ as the one associated with $\mu_{[k]}$."

Remark 1: In (3b) it is assumed that the experimenter has employed a completely randomized design in the first stage of experimentation; in fact, he must use

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[1/] For the special case in which the experimenter specifies $\delta_0^* = 0$, $\delta_1^* = \delta_2^* = \delta^*$ (say), the design constants (h,c) can be obtained from Table 1 in Section 6 which is entered with h and $\gamma = c/\delta^*$.

that design rather than (say) a randomized blocks design, for were he to use the latter design it would turn out that

$$S^2 = \sum_{i=1}^{k} \sum_{j=1}^{N_0} (X_{ij} - \sum_{j=1}^{N_0} X_{ij}/N_0 - \sum_{i=1}^{k} X_{ij}/k + \sum_{i=1}^{k} \sum_{j=1}^{N_0} X_{ij}/kN_0)^2/n$$

based on $n = (k-1)(N_0-1)$ d.f. will underestimate the variance associated with the total experiment. This is so because the final inference (3f) is made based on comparisons with the fixed known standard rather than on contrasts among the $\bar{x}_i$. (The situation here is different than that in B-D-S (1954).)

The selection procedure (3) is completely defined once values of the design constants (h,c) are assigned; as noted above, these depend on k,n and the specified quantities $\{\delta_0^*,\delta_1^*,\delta_2^*;P_0^*,P_1^*\}$. Theorem 1, stated and proved in the next section, tells how to determine (h,c) so as to guarantee (2a) and (2b).

In closing this section we emphasize that the total sample size N is a random variable (since S^2 is a random variable) its distribution depending not only on k,n and $\{\delta_0^*,\delta_1^*,\delta_2^*;P_0^*,P_1^*\}$ but also on σ^2; the variance of the distribution of S^2 decreases rapidly with increasing $n = k(N_0-1)$.

4. Theorems and proofs of theorems

4.1 Statement and proof of Theorem 1

Theorem 1: The (h,c) which guarantee (2a) and (2b) are the pair which satisfy the simultaneous equations:

$$\int_0^\infty \Phi^k(hz)g_n(z)dz = P_0^* \tag{4a}$$

$$\int_0^\infty \left[\int_{(c-\delta_1^*)hz/(c+\delta_0^*)}^\infty \Phi^{k-1}(y + \delta_2^* hz/(c + \delta_0^*))\phi(y)dy\right]g_n(z)dz = P_1^* \tag{4b}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal distribution and density function, respectively, and $g_n(\cdot)$ is the density function of $(\chi_n^2/n)^{1/2}$.

In many applications it will be sufficient to consider a specialization of (4). This is done in (5a,b).

Of special interest is the case $\delta_0^* = 0$, $\delta_1^* = \delta_2^* = \delta^*$ (say). Letting $\gamma = c/\delta^*$ we then have that (h^*,γ^*), $h^* = h_n^*(k,P_0^*,P_1^*)$, $\gamma^* = \gamma_n^*(k,P_0^*,P_1^*)$ are the pair which satisfy the simultaneous equations:

$$\int_0^\infty \Phi^k(hz)g_n(z)dz = P_0^* \tag{5a}$$

$$\int_0^\infty \left[\int_{(\gamma-1)hz/\gamma}^\infty \Phi^{k-1}(y + hz/\gamma)\phi(y)dy\right]g_n(z)dz = P_1^*, \tag{5b}$$

and $N = \max\{[(hS/\gamma\delta^*)^2] + 1, N_0\}$. Tables of (h^*,γ^*) for $k = 1(1)6,10$ and selected n and $\{P_0^*,P_1^*\}$ are given in Section 6. An example of the use of the tables is given in Section 7.

Proof of Theorem 1

We first consider (2a) using (3), and find that

$$\Pr\{\Pi_0|\mu_{[k]} \leq \mu_0 - \delta_0^*\} = \Pr\{\overline{X}_{[k]} < \mu_0 + c|\mu_{[k]} \leq \mu_0 - \delta_0^*\}$$

$$= \Pr\{\overline{X}_i - \mu_i < \mu_0 - \mu_i + c\ (1 \leq i \leq k)|\mu_i \leq \mu_0 - \delta_0^*\ (1 \leq i \leq k)\}$$

$$\geq \Pr\left\{\frac{\overline{X}_i - \mu_i}{\sqrt{S^2/N}} < \frac{\delta_0^* + c}{\sqrt{S^2/N}}\ (1 \leq i \leq k)\right\}. \tag{6a}$$

We proceed by first conditioning on S^2, noting that conditional on S^2 the distribution of $\overline{X}_i$ is $N(\overline{X}_i|\mu_i, \sigma^2/N)$, and unconditioning on S^2 (the distributions of $\overline{X}_i$ and S^2 being independent when N is fixed). Continuing, we see that the r.h.s. of (6a) is no less than

$$\Pr\left\{\frac{\overline{X}_i - \mu_i}{\sqrt{S^2/N}} < h_0 \quad (1 \leq i \leq k)\right\}$$

$$= \Pr\left\{\frac{\overline{X}_i - \mu_i}{\sqrt{\sigma^2/N}} < h_0\sqrt{\frac{S^2}{\sigma^2}} \quad (1 \leq i \leq k)\right\}$$

$$= \int_0^\infty \Phi^k(h_0 z) g_n(z)\,dz \tag{6b}$$

provided that

$$N \geq \left(\frac{S}{\delta_0^* + c}\right)^2 h_0^2. \tag{7}$$

We next consider (2b) using (3), and find (using the same conditioning argument as above, and the monotonicity results proved in B-T (1974)) that

$$\Pr\{\Pi_{[k]}|\mu_{[k]} \geq \mu_0 + \delta_1^*,\ \mu_{[k]} \geq \mu_{[k-1]} + \delta_2^*\}$$

$$= \Pr\left\{\overline{X}_{(k)} > \mu_0 + c,\ \overline{X}_{(i)} < \overline{X}_{(k)}\ (1 \leq i \leq k-1) \,\middle|\, \begin{matrix}\mu_{[k]} \geq \mu_0 + \delta_1^* \\ \mu_{[k]} \geq \mu_{[k-1]} + \delta_2^*\end{matrix}\right\}$$

$$\geq \Pr\left\{\frac{\overline{X}_{(k)} - \mu_{[k]}}{\sqrt{S^2/N}} > \frac{\overline{X}_{(i)} - \mu_{[i]}}{\sqrt{S^2/N}} - \frac{\delta_2^*}{\sqrt{S^2/N}} \quad (1 \leq i \leq k-1),\right.$$

$$\left.\frac{\overline{X}_{(k)} - \mu_{[k]}}{\sqrt{S^2/N}} > \frac{c - \delta_1^*}{\sqrt{S^2/N}}\right\} \tag{8a}$$

where $\overline{X}_{(i)}$ denotes the over-all sample mean based on N independent observations from the population having mean $\mu_{[i]}$ $(1 \leq i \leq k)$. Continuing we see that the r.h.s. of (8a) is no less than

$$\Pr\left\{\frac{\overline{X}_{(k)} - \mu_{[k]}}{\sqrt{S^2/N}} > \frac{\overline{X}_{(i)} - \mu_{[i]}}{\sqrt{S^2/N}} - h_2 \ (1 \leq i \leq k-1), \ \frac{\overline{X}_{(k)} - \mu_{[k]}}{\sqrt{S^2/N}} > -h_1\right\}$$

$$= \int_0^\infty \left[\int_{-h_1 z}^\infty \Phi^{k-1}(y + h_2 z)\phi(y)dy\right] g_n(z)dz \tag{8b}$$

provided that

$$N \geq \max\{(Sh_1/(\delta_1^* - c))^2, \ (Sh_2/\delta_2^*)^2\}. \tag{9}$$

Combining (7) and (9) we see that we must have

$$N \geq S^2 \cdot \max\{(h_0/(\delta_1^* + c))^2, \ (h_1/(\delta_1^* - c))^2, \ (h_2/\delta_2^*)^2\}. \tag{10}$$

Now we will be choosing h_0, h_1, h_2, c so that the probabilities (6b), (8b) are equal to P_0^*, P_1^*, respectively. Subject to these conditions we see that the r.h.s. of (10) is minimized when

$$h_0^2/(\delta_1^* + c)^2 = h_1^2/(\delta_1^* - c)^2 = (h_2/\delta_2^*)^2.$$

Thus, denoting h_0 by h we find that if

$$h_1 = h(\delta_1^* - c)/(\delta_0^* + c), \ h_2 = h\delta_2^*/(\delta_0^* + c) \tag{11}$$

we will be able to select the most economical sample size N. Substituting this minimax choice of h_1, h_2 in the expressions (6b), (8b), and equating the resulting integrals to P_0^*, P_1^*, respectively, we obtain (4a), (4b), respectively. This completes the proof.

Corollary 1: For $n \to \infty$, the pair $h\ (= h_\infty^*)$, $c\ (= c_\infty^*)$ which guarantee (2a) and (2b) are the pair which satisfy the simultaneous equations:

$$\Phi^k(h) = P_0^* \tag{12a}$$

$$\int_{(c-\delta_1^*)h/(c+\delta_0^*)}^{\infty} \Phi^{k-1}(y + \delta_2^* h/(c+\delta_0^*))\phi(y)dy = P_1^*, \tag{12b}$$

and if $P_0^* > 2^{-k}$, $P_1^* > (1-2^{-k})/k$, then $h_\infty^* > 0$ and $-\delta_0^* < c_\infty^* < \delta_1^*$.

Proof: Obvious, since $S^2 \to \sigma^2$ w.p. 1.

Remark 2. The case $n \to \infty$ corresponds to the case when σ^2 is known. When this is so, the probability requirements (2a,b) are clearly satisfied by taking $N_0 = N = \sigma^2(h_\infty^*/(\delta_0^* + c_\infty^*))^2$, ignoring the integer condition on N. With this substitution the equations (12a,b) reduce exactly to the equations (4a,b) of Bechhofer and Turnbull [1974].

4.2 Statement and proof of Theorem 2

Theorem 2: The same (h,c) which guarantee (2a,b) when $\delta_1^* = \delta_2^* = \delta^*$ (say), also guarantee

$$\Pr\{\Pi_{[k]} \text{ or } \Pi_{[k-1]} \text{ or } \cdots \text{ or } \Pi_{[k-t+1]}\} \geq P_1^*$$

whenever

a) $\mu_{[k]} \geq \mu_0 + \delta^*, \quad \mu_{[k-t+1]} \geq \mu_0 \geq \mu_{[k-t]}$

or

b) $\mu_{[k]} \geq \mu_0 + \delta^*, \quad \mu_{[k-t+1]} \geq \mu_{[k-t]} + \delta^*,$

for any t $(1 \leq t \leq k)$, the probability being strict if $t > 1$; here the event $\Pi_{[i]}$ $(1 \leq i \leq k)$ corresponds to selecting the population associated with $\mu_{[i]}$, and we define $\mu_{[0]} = -\infty$.

Proof of Theorem 2

The proof follows the same lines as those given for Theorem 2 in B-T (1974), and thus is not given here.

5. The expected sample size and the choice of N_0

Following Stein (1945), we have

$$E\{N\} = N_0 \Pr\{\chi_n^2 < nN_0 z^2/\sigma^2\}$$
$$+ (\sigma^2/z^2)\Pr\{\chi_{n+2}^2 > nN_0 z^2/\sigma^2\} + \theta \cdot \Pr\{\chi_n^2 > nN_0 z^2/\sigma^2\} \qquad (13)$$

where $z = (\delta_0^* + c)/h$ and $0 \leq \theta < 1$. A similar expression can be derived for $E\{N^2\}$. (An expression analogous to (13) has been tabulated in Tables I-IV of Seelbinder (1953); his $c = d/\sigma$, t, n_0 correspond to our $(\delta_0^* + c)/\sigma$, h, $k(N_0 - 1)$, respectively; however, his tables are applicable here only when $k = 1$.)

If the experimenter has some idea as to the possible values of σ, this information can be used to assist in the choice of N_0. For example, suppose that it can be reasonably assumed that σ lies in the range $[\sigma_1, \sigma_2]$. Using the minimax regret criterion of Seelbinder (1953), N_0 would be chosen to

minimize the maximum expected loss in number of extra observations needed due to ignorance of σ, i.e., N_0 minimizes

$$\max_{\sigma_1 \leq \sigma \leq \sigma_2} \left(E\{N\} - (\sigma h_\infty^* / (\delta_0^* + c_\infty^*))^2\right), \tag{14}$$

where h_∞^*, c_∞^* are the solutions of (12a,b). In (14), $E\{N\}$ of course depends on N_0 and σ, while by Remark 2 in Section 4.1 the second term represents the total sample size (within unity) if σ were known. For the special case $\delta_0^* = 0$, $\delta_1^* = \delta_2^* = \delta^*$, the value of h_∞^* and c_∞^* $(= \gamma_\infty^* \delta^*)$ can be obtained from the bottom line of the appropriate column in the tables of Section 6, or, alternatively, from the tables in Bechhofer and Turnbull (1974). (There, quantities λ_1^*, λ_2^* are tabulated; these are related to h_∞^*, γ_∞^* by $h_\infty^* = \lambda_1^*$, $\gamma_\infty^* = \lambda_1^*/\lambda_2^*$.)

It is clear that the variance of N increases as N_0 decreases. Hence, as an alternative to the above procedure for choosing N_0, we may wish to increase N_0 (perhaps by only a small number of observations if k is large) in order to reduce the probability of being required to take an extremely large N; this gain is purchased at the cost of a slight increase in $E\{N\}$. Moshman (1958) has discussed such criteria in a similar two-stage problem, and his methods are applicable here.

Finally, it may be that there is a specified upper bound N^* on the total sample size N. If the procedure calls for more than N^* observations, then N^* observations are taken, and the probabilities P_0^*, P_1^* are reduced. Such a procedure can be legitimately constructed by using methods similar to those of Wormleighton (1960).

6. Tables

Table 1 gives the solution (h^*,γ^*) of equations (5a,b) for $k = 1(1)6,10$ and selected n and $\{P_0^*,P_1^*\}$. The tabulated values of (h^*,γ^*) are calculated to an accuracy of $\pm 10^{-4}$ in the associated (P_0^*,P_1^*)-values.

7. A numerical example of the use of the tables

A consumer is to decide whether or not to purchase one lot of bolts from among three lots which are being offered for his consideration. The tensile strength (X) of the bolts in the ith lot $(1 \leq i \leq 3)$ may be assumed to be normally distributed with unknown variance σ^2 and unknown mean μ_i. A lot is deemed acceptable only if the bolts in the lot have a mean tensile strength of at least 60,000 psi. Suppose that he asks for a procedure which will guarantee $P_0^* = 0.95$ and $P_1^* = 0.90$ with $\delta_0^* = 0$, $\delta_1^* = \delta_2^* = 250$ psi and $\mu_0 = 60{,}000$ psi. How should we proceed to guarantee his requirement?

The choice of $N_0 \geq 2$ is optional. Suppose that we take a preliminary sample of $N_0 = 16$ observations (bolts) from each lot, and using (3b) we obtain the estimate $S^2 = (753.1 \text{ psi})^2$ of σ^2 based on 45 d.f. Entering Table 1 with $k = 3$, $n = 45$, $P_0^* = 0.95$, $P_1^* = 0.90$ we find that $h^* = 2.1855$ and $\gamma^* = 0.6234$. Hence, $c = (0.6234)(250) = 156$ psi, and from (3d) we see that $N = \max\{[\{(2.1855)(753.1)/(0.6234)(250)\}^2] + 1, 16\} = 112$. Therefore 96 additional observations (bolts) are taken from each lot, and the three sample means (based on all 112 observations per lot) are computed. If all of these sample means are less than $\mu_0 + c = 60{,}156$ psi, then no lot is accepted; otherwise the lot that produced the largest sample mean is accepted.

8. Extensions and directions of future research

It is straightforward to extend the results in this paper to the case of known variance ratios, i.e., when it is assumed that the variance of population

<u>Table 1</u>

<u>Values of</u> h^* <u>and</u> γ^*

In the table, h^* is the upper entry and γ^* is the lower entry

K = 1

n	P_0^* =	0.75	0.90	0.90	0.95	0.95	0.95	0.99	0.99	0.99	0.99
	P_1^* =	0.75	0.75	0.90	0.75	0.90	0.95	0.75	0.90	0.95	0.99
1		1.0000 0.5000	3.0777 0.7548	3.0777 0.5000	6.3138 0.8633	6.3138 0.6723	6.3138 0.5000	31.8205 0.9695	31.8205 0.9118	31.8205 0.8344	31.8205 0.5000
2		0.8165 0.5000	1.8856 0.6978	1.8856 0.5000	2.9200 0.7815	2.9200 0.6076	2.9200 0.5000	6.9646 0.8951	6.9646 0.7869	6.9646 0.7046	6.9646 0.5000
3		0.7649 0.5000	1.6377 0.6816	1.6377 0.5000	2.3534 0.7547	2.3534 0.5897	2.3534 0.5000	4.5407 0.8558	4.5407 0.7349	4.5407 0.6586	4.5407 0.5000
4		0.7407 0.5000	1.5332 0.6743	1.5332 0.5000	2.1318 0.7421	2.1318 0.5817	2.1318 0.5000	3.7469 0.8349	3.7469 0.7096	3.7469 0.6374	3.7469 0.5000
5		0.7267 0.5000	1.4759 0.6701	1.4759 0.5000	2.0150 0.7350	2.0150 0.5772	2.0150 0.5000	3.3649 0.8224	3.3649 0.6951	3.3649 0.6255	3.3649 0.5000
7		0.7111 0.5000	1.4149 0.6655	1.4149 0.5000	1.8946 0.7271	1.8946 0.5725	1.8946 0.5000	2.9980 0.8083	2.9980 0.6794	2.9980 0.6128	2.9980 0.5000
9		0.7027 0.5000	1.3830 0.6631	1.3830 0.5000	1.8331 0.7229	1.8331 0.5700	1.8331 0.5000	2.8214 0.8006	2.8214 0.6711	2.8214 0.6062	2.8214 0.5000
11		0.6974 0.5000	1.3634 0.6616	1.3634 0.5000	1.7959 0.7203	1.7959 0.5684	1.7959 0.5000	2.7181 0.7958	2.7181 0.6659	2.7181 0.6021	2.7181 0.5000
13		0.6938 0.5000	1.3502 0.6606	1.3502 0.5000	1.7709 0.7185	1.7709 0.5674	1.7709 0.5000	2.6503 0.7925	2.6503 0.6625	2.6503 0.5994	2.6503 0.5000
15		0.6912 0.5000	1.3406 0.6598	1.3406 0.5000	1.7531 0.7172	1.7531 0.5667	1.7531 0.5000	2.6025 0.7901	2.6025 0.6600	2.6025 0.5975	2.6025 0.5000

K = 1

n	$P_0^* =$	0.75	0.90	0.90	0.95	0.95	0.95	0.99	0.99	0.99	0.99
	$P_1^* =$	0.75	0.75	0.90	0.75	0.90	0.95	0.75	0.90	0.95	0.99
18		0.6884 0.5000	1.3304 0.6590	1.3304 0.5000	1.7341 0.7158	1.7341 0.5659	1.7341 0.5000	2.5524 0.7876	2.5524 0.6574	2.5524 0.5955	2.5524 0.5000
21		0.6864 0.5000	1.3232 0.6585	1.3232 0.5000	1.7207 0.7149	1.7207 0.5653	1.7207 0.5000	2.5176 0.7858	2.5176 0.6555	2.5176 0.5940	2.5176 0.5000
24		0.6848 0.5000	1.3178 0.6580	1.3178 0.5000	1.7109 0.7141	1.7109 0.5649	1.7109 0.5000	2.4922 0.7844	2.4922 0.6541	2.4922 0.5929	2.4922 0.5000
27		0.6837 0.5000	1.3137 0.6577	1.3137 0.5000	1.7033 0.7136	1.7033 0.5646	1.7033 0.5000	2.4727 0.7834	2.4727 0.6530	2.4727 0.5921	2.4727 0.5000
30		0.6828 0.5000	1.3104 0.6575	1.3104 0.5000	1.6973 0.7131	1.6973 0.5643	1.6973 0.5000	2.4573 0.7826	2.4573 0.6522	2.4573 0.5915	2.4573 0.5000
33		0.6820 0.5000	1.3077 0.6572	1.3077 0.5000	1.6924 0.7128	1.6924 0.5641	1.6924 0.5000	2.4448 0.7819	2.4448 0.6515	2.4448 0.5909	2.4448 0.5000
36		0.6814 0.5000	1.3055 0.6571	1.3055 0.5000	1.6883 0.7125	1.6883 0.5639	1.6883 0.5000	2.4345 0.7813	2.4345 0.6509	2.4345 0.5905	2.4345 0.5000
39		0.6808 0.5000	1.3036 0.6569	1.3036 0.5000	1.6849 0.7122	1.6849 0.5638	1.6849 0.5000	2.4258 0.7808	2.4258 0.6505	2.4258 0.5901	2.4258 0.5000
42		0.6804 0.5000	1.3020 0.6568	1.3020 0.5000	1.6820 0.7120	1.6820 0.5637	1.6820 0.5000	2.4185 0.7804	2.4185 0.6500	2.4185 0.5898	2.4185 0.5000
45		0.6800 0.5000	1.3006 0.6567	1.3006 0.5000	1.6794 0.7118	1.6794 0.5636	1.6794 0.5000	2.4121 0.7801	2.4121 0.6497	2.4121 0.5895	2.4121 0.5000
∞		0.6745 0.5000	1.2816 0.6552	1.2816 0.5000	1.6449 0.7092	1.6449 0.5621	1.6449 0.5000	2.3263 0.7752	2.3263 0.6448	2.3263 0.5858	2.3263 0.5000

K = 2

n	P_0^* =	0.75	0.90	0.90	0.95	0.95	0.95	0.99	0.99	0.99	0.99
	P_1^* =	0.75	0.75	0.90	0.75	0.90	0.95	0.75	0.90	0.95	0.99
2		1.4523	2.7433	2.7433	4.0750	4.0750	4.0750	9.4515	9.4515	9.4515	9.4515
		0.6042	0.7602	0.5745	0.8291	0.6755	0.5677	0.9201	0.8327	0.7617	0.5630
4		1.2601	2.0722	2.0722	2.7215	2.7215	2.7215	4.5441	4.5441	4.5441	4.5441
		0.5991	0.7274	0.5632	0.7822	0.6342	0.5531	0.8593	0.7468	0.6792	0.5433
6		1.2053	1.9047	1.9047	2.4170	2.4170	2.4170	3.6843	3.6843	3.6843	3.6843
		0.5977	0.7176	0.5600	0.7674	0.6222	0.5486	0.8365	0.7183	0.6536	0.5369
8		1.1795	1.8294	1.8294	2.2849	2.2849	2.2849	3.3424	3.3424	3.3424	3.3424
		0.5972	0.7130	0.5584	0.7603	0.6166	0.5465	0.8250	0.7046	0.6416	0.5338
10		1.1645	1.7866	1.7866	2.2113	2.2113	2.2113	3.1606	3.1606	3.1606	3.1606
		0.5968	0.7103	0.5576	0.7562	0.6134	0.5453	0.8182	0.6967	0.6347	0.5319
14		1.1477	1.7399	1.7399	2.1321	2.1321	2.1321	2.9717	2.9717	2.9717	2.9717
		0.5965	0.7073	0.5566	0.7515	0.6098	0.5439	0.8105	0.6879	0.6272	0.5299
18		1.1386	1.7149	1.7149	2.0902	2.0902	2.0902	2.8747	2.8747	2.8747	2.8747
		0.5963	0.7057	0.5560	0.7490	0.6079	0.5432	0.8063	0.6831	0.6231	0.5264
22		1.1329	1.6993	1.6993	2.0643	2.0643	2.0643	2.8158	2.8158	2.8158	2.8158
		0.5962	0.7047	0.5557	0.7474	0.6067	0.5427	0.8037	0.6801	0.6206	0.5281
26		1.1290	1.6886	1.6886	2.0467	2.0467	2.0467	2.7762	2.7762	2.7762	2.7762
		0.5961	0.7039	0.5555	0.7463	0.6059	0.5424	0.8018	0.6781	0.6189	0.5276
30		1.1261	1.6809	1.6809	2.0339	2.0339	2.0339	2.7478	2.7478	2.7478	2.7478
		0.5961	0.7034	0.5553	0.7455	0.6053	0.5421	0.8005	0.6766	0.6177	0.5272
36		1.1230	1.6726	1.6726	2.0203	2.0203	2.0203	2.7175	2.7175	2.7175	2.7175
		0.5960	0.7029	0.5551	0.7446	0.6046	0.5419	0.7991	0.6750	0.6162	0.5269
42		1.1208	1.6667	1.6667	2.0106	2.0106	2.0106	2.6963	2.6963	2.6963	2.6963
		0.5960	0.7025	0.5550	0.7440	0.6041	0.5417	0.7981	0.6739	0.6154	0.5266
48		1.1192	1.6623	1.6623	2.0035	2.0035	2.0035	2.6806	2.6806	2.6806	2.6806
		0.5959	0.7022	0.5549	0.7435	0.6038	0.5416	0.7973	0.6731	0.6147	0.5264
∞		1.1078	1.6322	1.6322	1.9545	1.9545	1.9545	2.5750	2.5750	2.5750	2.5750
		0.5957	0.7002	0.5542	0.7404	0.6014	0.5406	0.7920	0.6673	0.6098	0.5250

K = 3

n										
	$P_0^* =$ 0.75	0.90	0.90	0.95	0.95	0.95	0.99	0.99	0.99	0.99
	$P_1^* =$ 0.75	0.75	0.90	0.75	0.90	0.95	0.75	0.90	0.95	0.99
3	1.6363 0.6413	2.6481 0.7639	2.6481 0.5997	3.5508 0.8179	3.5508 0.6760	3.5508 0.5878	6.4616 0.8936	6.4616 0.7966	6.4616 0.7307	6.4616 0.5772
6	1.4708 0.6370	2.1716 0.7409	2.1716 0.5889	2.6957 0.7853	2.6957 0.6461	2.6957 0.5734	4.0135 0.8477	4.0135 0.7350	4.0135 0.6723	4.0135 0.5569
9	1.4218 0.6358	2.0421 0.7338	2.0421 0.5856	2.4787 0.7748	2.4787 0.6370	2.4787 0.5689	3.4867 0.8316	3.4867 0.7152	3.4867 0.6543	3.4867 0.5503
12	1.3984 0.6353	1.9820 0.7304	1.9820 0.5840	2.3805 0.7697	2.3805 0.6326	2.3805 0.5667	3.2618 0.8236	3.2618 0.7056	3.2618 0.6457	3.2618 0.5471
15	1.3846 0.6350	1.9473 0.7284	1.9473 0.5830	2.3245 0.7667	2.3245 0.6300	2.3245 0.5653	3.1378 0.8189	3.1378 0.6999	3.1378 0.6407	3.1378 0.5452
21	1.3692 0.6346	1.9089 0.7261	1.9089 0.5819	2.2632 0.7633	2.2632 0.6272	2.2632 0.5639	3.0052 0.8135	3.0052 0.6937	3.0052 0.6352	3.0052 0.5430
27	1.3608 0.6345	1.8881 0.7248	1.8881 0.5814	2.2303 0.7614	2.2303 0.6256	2.2303 0.5631	2.9354 0.8105	2.9354 0.6902	2.9354 0.6321	2.9354 0.5418
33	1.3554 0.6343	1.8751 0.7241	1.8751 0.5810	2.2097 0.7603	2.2097 0.6246	2.2097 0.5626	2.8925 0.8086	2.8925 0.6881	2.8925 0.6303	2.8925 0.5411
39	1.3518 0.6343	1.8662 0.7235	1.8662 0.5807	2.1957 0.7594	2.1957 0.6239	2.1957 0.5622	2.8633 0.8073	2.8633 0.6866	2.8633 0.6290	2.8633 0.5406
45	1.3491 0.6342	1.8597 0.7231	1.8597 0.5805	2.1855 0.7589	2.1855 0.6234	2.1855 0.5620	2.8423 0.8064	2.8423 0.6855	2.8423 0.6280	2.8423 0.5402
∞	1.3319 0.6339	1.8183 0.7205	1.8183 0.5793	2.1212 0.7550	2.1212 0.6203	2.1212 0.5603	2.7119 0.8003	2.7119 0.6786	2.7119 0.6220	2.7119 0.5378

K = 4

n	P_0^* =	0.75	0.90	0.90	0.95	0.95	0.95	0.99	0.99	0.99	0.99
	P_1^* =	0.75	0.75	0.90	0.75	0.90	0.95	0.75	0.90	0.95	0.99
4		1.7507 0.6625	2.6201 0.7674	2.6201 0.6137	3.3401 0.8134	3.3401 0.6775	3.3401 0.5984	5.4118 0.8789	5.4118 0.7780	5.4118 0.7153	5.4118 0.5830
8		1.6065 0.6590	2.2433 0.7496	2.2433 0.6040	2.7035 0.7882	2.7035 0.6537	2.7035 0.5853	3.7990 0.8425	3.7990 0.7302	3.7990 0.6700	3.7990 0.5642
12		1.5626 0.6580	2.1357 0.7439	2.1357 0.6009	2.5310 0.7800	2.5310 0.6462	2.5310 0.5811	3.4134 0.8301	3.4134 0.7148	3.4134 0.6559	3.4134 0.5580
16		1.5413 0.6575	2.0849 0.7411	2.0849 0.5994	2.4511 0.7760	2.4511 0.6426	2.4511 0.5783	3.2427 0.8239	3.2427 0.7073	3.2427 0.6491	3.2427 0.5550
20		1.5288 0.6572	2.0553 0.7395	2.0553 0.5985	2.4051 0.7736	2.4051 0.6405	2.4051 0.5777	3.1467 0.8202	3.1467 0.7029	3.1467 0.6451	3.1467 0.5532
28		1.5147 0.6569	2.0222 0.7376	2.0222 0.5975	2.3541 0.7709	2.3541 0.6380	2.3541 0.5763	3.0425 0.8160	3.0425 0.6980	3.0425 0.6407	3.0425 0.5512
36		1.5069 0.6567	2.0042 0.7366	2.0042 0.5969	2.3265 0.7694	2.3265 0.6367	2.3265 0.5755	2.9870 0.8136	2.9870 0.6952	2.9870 0.6382	2.9870 0.5501
44		1.5020 0.6566	1.9929 0.7359	1.9929 0.5965	2.3092 0.7684	2.3092 0.6359	2.3092 0.5750	2.9526 0.8122	2.9526 0.6935	2.9526 0.6367	2.9526 0.5494
∞		1.4803 0.6562	1.9432 0.7330	1.9432 0.5949	2.2340 0.7642	2.2340 0.6321	2.2340 0.5728	2.8058 0.8056	2.8058 0.6860	2.8058 0.6299	2.8058 0.5462

K = 5

n	P_0^* =	0.60	0.75	0.75	0.90	0.90	0.90	0.95	0.95	0.95	0.95
	P_1^* =	0.60	0.60	0.75	0.60	0.75	0.90	0.60	0.75	0.90	0.95
5		1.4170	1.8340	1.8340	2.6159	2.6159	2.6159	3.2343	3.2343	3.2343	3.2343
		0.7504	0.8283	0.6768	0.8950	0.7705	0.6233	0.9188	0.8114	0.6793	0.6054
10		1.3553	1.7057	1.7057	2.2999	2.2999	2.2999	2.7205	2.7205	2.7205	2.7205
		0.7523	0.8250	0.6739	0.8863	0.7558	0.6145	0.9078	0.7907	0.6593	0.5935
15		1.3358	1.6658	1.6658	2.2068	2.2068	2.2068	2.5755	2.5755	2.5755	2.5755
		0.7530	0.8241	0.6730	0.8835	0.7511	0.6116	0.9041	0.7839	0.6529	0.5895
20		1.3262	1.6464	1.6464	2.1623	2.1623	2.1623	2.5074	2.5074	2.5074	2.5074
		0.7535	0.8236	0.6726	0.8821	0.7487	0.6102	0.9022	0.7806	0.6498	0.5876
25		1.3205	1.6349	1.6349	2.1362	2.1362	2.1362	2.4678	2.4678	2.4678	2.4678
		0.7537	0.8234	0.6723	0.8812	0.7473	0.6093	0.9011	0.7786	0.6479	0.5864
35		1.3140	1.6219	1.6219	2.1070	2.1070	2.1070	2.4237	2.4237	2.4237	2.4237
		0.7540	0.8231	0.6720	0.8802	0.7457	0.6084	0.8999	0.7763	0.6458	0.5851
45		1.3105	1.6148	1.6148	2.0910	2.0910	2.0910	2.3997	2.3997	2.3997	2.3997
		0.7542	0.8229	0.6719	0.8797	0.7449	0.6078	0.8992	0.7750	0.6446	0.5843
55		1.3082	1.6102	1.6102	2.0810	2.0810	2.0810	2.3847	2.3847	2.3847	2.3847
		0.7543	0.8228	0.6718	0.8794	0.7443	0.6075	0.8987	0.7742	0.6439	0.5839
∞		1.2981	1.5900	1.5900	2.0365	2.0365	2.0365	2.3187	2.3187	2.3187	2.3187
		0.7548	0.8223	0.6714	0.8778	0.7418	0.6059	0.8967	0.7707	0.6406	0.5818

K = 6

n	P_0^* =	0.60	0.75	0.75	0.90	0.90	0.90	0.95	0.95	0.95	0.95
	P_1^* =	0.60	0.60	0.75	0.60	0.75	0.90	0.60	0.75	0.90	0.95
6		1.5050	1.8998	1.8998	2.6214	2.6214	2.6214	3.1749	3.1749	3.1749	3.1749
		0.7624	0.8342	0.6874	0.8960	0.7732	0.6303	0.9180	0.8106	0.6810	0.6106
12		1.4480	1.7836	1.7836	2.3468	2.3468	2.3468	2.7401	2.7401	2.7401	2.7401
		0.7645	0.8316	0.6849	0.8887	0.7606	0.6223	0.9087	0.7929	0.6637	0.5996
18		1.4298	1.7470	1.7470	2.2642	2.2642	2.2642	2.6140	2.6140	2.6140	2.6140
		0.7653	0.8308	0.6841	0.8862	0.7565	0.6197	0.9056	0.7871	0.6581	0.5960
24		1.4208	1.7290	1.7290	2.2243	2.2243	2.2243	2.5541	2.5541	2.5541	2.5541
		0.7657	0.8305	0.6837	0.8850	0.7545	0.6183	0.9040	0.7842	0.6553	0.5941
30		1.4154	1.7184	1.7184	2.2009	2.2009	2.2009	2.5191	2.5191	2.5191	2.5191
		0.7660	0.8302	0.6835	0.8843	0.7533	0.6175	0.9030	0.7825	0.6536	0.5930
42		1.4093	1.7063	1.7063	2.1745	2.1745	2.1745	2.4800	2.4800	2.4800	2.4800
		0.7663	0.8300	0.6832	0.8835	0.7519	0.6166	0.9020	0.7805	0.6518	0.5918
54		1.4060	1.6996	1.6996	2.1601	2.1601	2.1601	2.4587	2.4587	2.4587	2.4587
		0.7665	0.8299	0.6831	0.8830	0.7511	0.6161	0.9014	0.7794	0.6507	0.5911
66		1.4038	1.6954	1.6954	2.1510	2.1510	2.1510	2.4453	2.4453	2.4453	2.4453
		0.7666	0.8298	0.6830	0.8827	0.7506	0.6158	0.9010	0.7787	0.6501	0.5907
∞		1.3943	1.6765	1.6765	2.1105	2.1105	2.1105	2.3862	2.3862	2.3862	2.3862
		0.7671	0.8294	0.6826	0.8814	0.7484	0.6143	0.8992	0.7756	0.6471	0.5887

K = 10

n	P_0^* =	0.60	0.75	0.75	0.90	0.90	0.90	0.95	0.95	0.95	0.95
	P_1^* =	0.60	0.60	0.75	0.60	0.75	0.90	0.60	0.75	0.90	0.95
10		1.7347	2.0765	2.0765	2.6693	2.6693	2.6693	3.0960	3.0960	3.0960	3.0960
		0.7910	0.8489	0.7130	0.8995	0.7813	0.6477	0.9176	0.8109	0.6870	0.6233
20		1.6904	1.9896	1.9896	2.4814	2.4814	2.4814	2.8158	2.8158	2.8158	2.8158
		0.7933	0.8477	0.7114	0.8949	0.7731	0.6417	0.9117	0.7995	0.6753	0.6149
30		1.6758	1.9613	1.9613	2.4223	2.4223	2.4223	2.7297	2.7297	2.7297	2.7297
		0.7941	0.8473	0.7108	0.8934	0.7703	0.6396	0.9097	0.7957	0.6714	0.6121
40		1.6685	1.9472	1.9472	2.3933	2.3933	2.3933	2.6880	2.6880	2.6880	2.6880
		0.7945	0.8471	0.7105	0.8926	0.7690	0.6385	0.9086	0.7938	0.6695	0.6106
50		1.6642	1.9388	1.9388	2.3761	2.3761	2.3761	2.6634	2.6634	2.6634	2.6634
		0.7947	0.8469	0.7104	0.8921	0.7681	0.6379	0.9080	0.7926	0.6683	0.6098
70		1.6592	1.9293	1.9293	2.3566	2.3566	2.3566	2.6356	2.6356	2.6356	2.6356
		0.7950	0.8468	0.7102	0.8916	0.7672	0.6372	0.9073	0.7913	0.6670	0.6088
90		1.6564	1.9240	1.9240	2.3459	2.3459	2.3459	2.6203	2.6203	2.6203	2.6203
		0.7952	0.8467	0.7101	0.8913	0.7666	0.6368	0.9069	0.7906	0.6662	0.6082
110		1.6547	1.9206	1.9206	2.3391	2.3391	2.3391	2.6107	2.6107	2.6107	2.6107
		0.7953	0.8467	0.7100	0.8911	0.7663	0.6365	0.9067	0.7901	0.6658	0.6079
∞		1.6468	1.9055	1.9055	2.3087	2.3087	2.3087	2.5679	2.5679	2.5679	2.5679
		0.7958	0.8465	0.7097	0.8902	0.7648	0.6353	0.9056	0.7880	0.6637	0.5063

Π_i $(1 \leq i \leq k)$ is $\sigma_i^2 = a_i\sigma^2$ where the a_i are known and σ^2 is unknown; the methods would be analogous to that used in B-D-S (1954).

It would be desirable to devise two-stage (or multi-stage) procedures when the a_i are also unknown. Presumably the methods of Dudewicz and Dalal (1971) or of Rinott (1974) could be extended to deal with this problem.

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1. REPORT NUMBER 256	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A (k+1)-DECISION TWO-STAGE SELECTION PROCEDURE FOR COMPARING k NORMAL MEANS WITH A FIXED KNOWN STANDARD: THE CASE OF COMMON UNKNOWN VARIANCE		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Robert E. Bechhofer Bruce W. Turnbull		8. CONTRACT OR GRANT NUMBER(s) DAHC04-73-C-0008 N00014-67-A-0077-0020
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Operations Research College of Engineering, Cornell University Ithaca, New York 14853		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Sponsoring Military Activity U.S. Army Research Office Durham, N.C. 27706		12. REPORT DATE May 1975 13. NUMBER OF PAGES 22
14. CONTROLLING OFFICE NAME AND ADDRESS Sponsoring Military Activity Statistics and Probability Program Office of Naval Research Arlington, Virginia 22217		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Ranking procedures, selection procedures, two-stage procedures, comparisons with a fixed standard, indifference-zone approach, sampling plans

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A (k+1)-decision two-stage selection procedure is proposed for the problem of comparing k normal means with a fixed known standard when the associated populations have a common unknown variance. None or one of the populations is to be selected, the procedure having the property that i) with probability at least P_0^* (specified), no population is to be selected when the largest

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

population mean is sufficiently less than the standard, and ii) with probability at least P_1^* (specified), the population with the largest population mean is to be selected when that mean is sufficiently greater than its closest competitor and the standard. Tables to implement the procedure are provided. Applications and generalizations are described.